Pascal’s Principle

After you study each sample problem and solution, work out the practice problems on a separate sheet of paper. Write your answers in the spaces provided.

Problem

A dentist’s chair makes use of Pascal’s principle to move patients up and down. Together, the chair and a patient exert a downward force of 2269 N. The chair is attached to a large piston with an area of 1221 cm². To move the chair, a pump applies force to a small piston with an area of 88.12 cm². What force must be exerted on the small piston to lift the chair?

Solution

Step 1: List the given and unknown values.

Given:  
\[ F_2 = 2269 \text{ N} \]
\[ A_1 = 88.12 \text{ cm}^2 \]
\[ A_2 = 1221 \text{ cm}^2 \]

Unknown:  
\[ F_1 \]

Step 2: Write the equations for Pascal’s principle and pressure, force, and area.

\[ p_1 = p_2 \]
\[ \text{pressure} = \frac{\text{force}}{\text{area}} \]

Step 3: Substitute force and area into the first equation, and rearrange for the desired value.

\[ p_1 = p_2 \]
\[ F_1 = \frac{F_2}{A_2} \]
\[ A_1 = A_2 \]
\[ F_1 = \frac{(F_2)(A_1)}{A_2} \]

Step 4: Insert the known values into the equation, and solve.

\[ F_1 = \frac{(2269 \text{ N})(88.12 \text{ cm}^2)}{1221 \text{ cm}^2} \]
\[ F_1 = 163.8 \text{ N} \]
Math Skills continued

Practice

1. A hydraulic lift office chair has its seat attached to a piston with an area of 11.2 cm². The chair is raised by exerting force on another piston, with an area of 4.12 cm². If a person sitting on the chair exerts a downward force of 219 N, what force needs to be exerted on the small piston to lift the seat?

2. In changing a tire, a hydraulic jack lifts 7468 N on its large piston, which has an area of 28.27 cm². How much force must be exerted on the small piston if it has an area of 1.325 cm²?

3. An engine shop uses a lift to raise a 1784 N engine. The lift has a large piston with an area of 76.32 cm². To raise the lift, force is exerted on a small piston with an area of 12.56 cm². What force must be exerted to raise the lift?

Problem

An engineering student wants to build her own hydraulic pump to lift a 1815 N crate. The pump will have two pistons connected via a fluid chamber. The student calculates that she will be able to exert 442 N of force on the small piston, which will have an area of 50.2 cm². What area must the large piston be to exert the desired force?

Solution

Step 1: List the given and unknown values.

Given:  
\[ F_1 = 442 \text{ N} \]
\[ A_1 = 50.2 \text{ cm}^2 \]
\[ F_2 = 1815 \text{ N} \]

Unknown:  
\[ A_2 \]

Step 2: Write the equations for Pascal's principle and pressure, force, and area.

\[ p_1 = p_2 \]
\[ \text{pressure} = \frac{\text{force}}{\text{area}} \]

Step 3: Substitute force and area into the first equation, and rearrange for the desired value.

\[ p_1 = p_2 \]
\[ \frac{F_1}{A_1} = \frac{F_2}{A_2} \]
\[ A_2 = \frac{F_2(A_1)}{F_1} \]
Step 4: Insert the known values into the equation, and solve.

\[ A_2 = \frac{(1815 \text{ N})(50.2 \text{ cm}^2)}{442 \text{ N}} \]
\[ A_2 = 206 \text{ cm}^2 \]

Practice

4. In a newly designed car with a hydraulic braking system, a force of 85 N is applied to one of the master cylinders, which has an area of 8.1 cm². The master cylinder is connected to one brake piston, which exerts a force of 296 N. What is the area of the brake piston?

5. A mechanic uses a hydraulic car jack to lift the front end of a car to change the oil. The jack she uses exerts 8915 N of force from the larger piston. To pump the jack, she exerts 444 N of force on the small piston, which has an area of 3.14 cm². What is the area of the large piston?

6. A student in the lunchroom blows into his straw with a force of 0.26 N. The column of air pushing the liquid in the glass has an area of 0.21 cm². If the liquid in the glass pushes upward with a force of 79 N, what is the area of the liquid at the surface of the glass?

Problem

The motor on a construction grade hydraulic shovel exerts $3.11 \times 10^7$ Pa of pressure on a fluid tank. The fluid tank is connected to a piston that has an area of 153 cm². How much force does the piston exert?

Solution

Step 1: List the given and unknown values.

**Given:**
- \( p_1 = 3.11 \times 10^7 \text{ Pa} \)
- \( A_2 = 153 \text{ cm}^2 \)

**Unknown:**
- \( F_2 \)

Step 2: Write the equations for Pascal’s principle and pressure, force, and area.

\[ p_1 = p_2 \]
\[ \text{pressure} = \frac{\text{force}}{\text{area}} \]
Math Skills continued

Step 3: Substitute force and area into the first equation, and rearrange for the desired value.

\[ p_1 = p_2 \]
\[ p_1 = \frac{F_2}{A_2} \]
\[ F_2 = (p_1)(A_2) \]

Step 4: Insert the known values into the equation, and solve.

\[ F_2 = (3.11 \times 10^7 \text{ Pa})(153 \text{ cm}^2) \]
\[ F_2 = \left( \frac{3.11 \times 10^7 \text{ N}}{\text{m}^2} \right)(1.53 \times 10^{-2} \text{ m}^2) \]
\[ F_2 = 4.76 \times 10^5 \text{ N} \]

Practice

7. A small crane has a motor that exerts $2.41 \times 10^7$ Pa of pressure on a fluid chamber. The chamber is connected by a fluid line to a piston on the crane arm. If the piston has an area of 168 cm$^2$, how much force does the piston exert?

8. A bicycle pump uses Pascal’s law to operate. The air in the hose acts as a fluid and transfers force and pressure from the piston to the tire. If a pump has a piston with an area of 7.1 cm$^2$, how much force must be exerted on it to create a pressure of $8.2 \times 10^5$ Pa?

9. A small, backyard log splitter has an engine that applies $1.723 \times 10^7$ Pa of pressure to a fluid tank. The tank is connected to piston with an area of 81.07 cm$^2$. How much force can the piston exert?

Mixed Practice

10. A force of 38.7 N is applied to the master cylinder of a hydraulic brake system. The cylinder has an area of 7.61 cm$^2$. The force from the master cylinder is transferred, by brake fluid, to two brake cylinders that have a total area of 49.1 cm$^2$. How much total force is exerted by the brake cylinders?

11. A factory lift is used to raise a load of 2225 N on a piston that has an area of 706.8 cm$^2$. How much pressure does the lift’s engine need to exert on the hydraulic fluid to lift the required load?
Bellringer Transparencies

SECTION: MATTER AND ENERGY
1. a. gas, molecules only
   b. gas, atoms only
   c. liquid, atoms only
   d. gas, atoms and molecules
   e. liquid, atoms and molecules
   f. solid, atoms and molecules
   g. solid, atoms only
   h. liquid, molecules only
   i. gas, atoms only
2. d, i

SECTION: FLUIDS
1. less; The buoyant force cannot equal the gold's weight, so the gold sinks.
2. greater; The buoyant force exceeds the balloon's weight, so the balloon rises.
3. equal; The weight of the boat is balanced by the buoyant force.
4. equal; The weight of the submarine is balanced by the buoyant force at the depth at which the submarine cruises.

SECTION: BEHAVIOR OF GASES
1. a
2. b
3. a

Concept Reviews

SECTION: MATTER AND ENERGY
1. a. liquid b. gas c. solid d. plasma
2. a. added b. fastest c. vaporization/evaporation d. absorbed e. slow down f. condensation g. released
3. The sugar molecules will have a lower speed on average than the water molecules because the sugar molecules are more massive than the water molecules. As the temperature of the mixture increases, the speed of all of the molecules will increase.
4. The total number of water molecules, and therefore the mass, stays the same; the molecules are just spread out over a greater volume. Energy is transferred from the surroundings to the water, so the water molecules are moving faster than they were, but the total amount of energy is the same.

SECTION: FLUIDS
1. An object in a fluid medium displaces a set amount of fluid upon immersion. Archimedes’ principle states that the weight of the displaced fluid is equal to the buoyant force exerted on the object.
2. The buoyant force exerted on the wood is equal to the weight of the wood.
3. 150 000 Pa
4. A fluid in equilibrium contained in a vessel exerts a pressure of equal intensity in all directions.
5. 33 250 N
6. As the speed of a moving fluid increases, its pressure decreases.

SECTION: BEHAVIOR OF GASES
1. a) Boyle's law, b) Charles's law, c) Gay-Lussac’s law
2. Solids have molecules fixed in relation to each other. Liquids have molecules capable of sliding past each other, but still stack together. Gases have molecules that are rarely in contact with each other. Also, solids have definite volume and shape. Liquids have definite volume and varying shape. Gases have varying shape and volume.
3. a
4. c
5. d
6. Either its pressure or volume must also change. Alternatively, both may change. The amplitude and direction of the changes depends on the original temperature change.

Math Skills

PASCAL’S PRINCIPLE
1. 80.6 N
2. 350.0 N
3. 293.6 N
4. 28 cm²
5. 63.0 cm²
6. 64 cm²
7. 4.05 \times 10^5 \text{ N}
8. 5.8 \times 10^2 \text{ N}
9. 1.397 \times 10^5 \text{ N}
10. 250 \text{ N}
11. 3.148 \times 10^4 \text{ Pa}

BOYLE’S LAW
1. 56.1 \text{ kPa}
2. 50.9 \text{ kPa}
3. 1.34 \times 10^4 \text{ kPa}
4. 449 \text{ kPa}
5. 711 \text{ L}
6. 165 \text{ L}
7. 378 \text{ L}
8. 0.262 \text{ L}
9. 3.6 \text{ L}
10. 101 \text{ L}
11. 370 \text{ kPa}
12. 5478 \text{ kPa}
13. 70 \text{ L}

Cross-Disciplinary

INTEGRATING BIOLOGY: DENSITY AND SWIM BLADERS
1. Less dense objects will float on more dense objects.
2. It allows to fish to hover, not requiring it to expend energy to maintain its position in the water.
3. By wearing just the right weight of lead, a scuba diver can adjust his or her density to equal that of the surrounding water so that the diver does not rise or sink in the water.

INTEGRATING PHYSICS: PLASMA
1. Plasma is a mixture of positive ions and free electrons; three examples are the interior of stars, neon signs, and the aurora borealis and australis.
2. Although we usually observe matter in its solid, liquid, and gaseous forms on Earth, most matter in the rest of the universe, including the interiors of stars (which are much more numerous than planets like Earth), is in the plasma state.
3. Plasma is considered to be a state of matter because its properties are very different from the properties of gases. Unlike a gas, a plasma is made of free electrons and positively charged ions.

INTEGRATING SPACE SCIENCE: OUR CHANGING UNIVERSE
1. Huge amounts of radiation prevented protons, neutrons, and electrons from binding together immediately after the big bang.
2. Light atoms such as hydrogen, hydrogen’s isotopes deuterium and tritium, helium, and lithium were the first to form.
3. Light elements were formed when protons, neutrons, and electrons from the big bang explosion joined together; heavier elements are formed in nuclear reactions that occur in stars.

REAL WORLD APPLICATIONS: GAS LAWS
1. Boyle’s Law relates the pressure and volume of a gas. Charles’s/Gay-Lussac’s Law relates the volume and pressure, respectively, of a gas to its temperature.
2. The pressure increases.
3. Heating the air inside a balloon increases its volume, making it less dense than surrounding air. This produces a buoyant force on the balloon.

REAL WORLD APPLICATIONS: SUBMARINES
1. Buoyancy is an upward force produced by the pressure of a fluid on an object.
2. The buoyant force on an object is equal to the weight of the fluid displaced by the object.
3. To make a submarine sink, sea water is pumped into the ballast tanks, making the submarine heavier than the water it displaces. The submarine rises when air replaces the water in the tanks.

SCIENCE AND THE CONSUMER: DRY ICE
1. The snow is subliming, or turning directly from a solid to a gas.
2. Carbon dioxide gas is more dense than air, so it sinks in air.
3. The dry ice keeps the ice cream frozen for a long period of time. The dry ice doesn’t melt, which would make the cardboard in the carton get wet.